

# Polymer Chemistry

## Macromolecules

Macromolecules also known as polymers, consist of a large no. of simple monomeric structural unit which repeated over and over again to form a giant like molecule by the covalent linkage b/w the repeating repeating monomers.

For example in the polymer  $-A-A-A-A-A-$  and  $-A-B-A-B-A-B-$  for instance, the repeated units are A and A-B respectively

e.g. Polyethylene



## Degree of polymerisation

The degree of polymerisation represents the no. of structural or monomeric units contained in a polymer. It is generally represented by  $K$  or  $\overline{DP}_n$

$$K = \frac{M_p}{M_0}$$

$M_p$  = mass of polymer

$M_0$  = " " monomer

## Extent of polymerisation

It is the fraction of monomer reacted at a given time i.e. it is the ratio of no. of molecules undergoing reaction to the initial no. of molecules. It is denoted by  $p$ .

$$\text{So, } p = \frac{N_0 - N}{N_0}$$

$$\Rightarrow p = 1 - \frac{N}{N_0}$$

$$\Rightarrow \frac{N_0}{N} = \frac{1}{1-p}$$

$N_0$  = no. of total monomer / initial no. of molecules

$N$  = no. of monomer at a given time / The no. of molecules

which does not undergo reaction.

## Molar masses of Polymers

The molar mass of a polymer increases continuously during the condensation reaction involved depending upon the degree of polymerisation. However, since the polymerisation chains might be broken at different stages, the final pdt generally contains macromolecules of different masses. It is necessary, therefore to take an avg. molar masses in these substances. To describe the distribution of molar masses, the following avg. are commonly used.

### i) Number avg. molar mass ( $\bar{M}_n$ )

It is defined as the obtained by carrying out the summation over the fraction of molecules multiplied by their corresponding molar mass, i.e.

$$\bar{M}_n = \sum_i f_i M_i$$

Subscript n stands for no. and f is as

$$f_i = \frac{N_i}{N_{\text{total}}}$$

Where  $N_i$  is the no. of molecules each having molar mass  $M_i$  and  $N_{\text{total}}$  is the total no. of molecules given by

$$N_{\text{total}} = \sum_i N_i$$

$$\text{So, } \bar{M}_n = \frac{\sum_i N_i M_i}{\sum_i N_i}$$

$N_i$  = no. of i type molecule per unit vol<sup>n</sup>

$$= \frac{N_1 M_1 + N_2 M_2 + N_3 M_3 + \dots}{N_1 + N_2 + N_3 + \dots}$$

### ii) Mass avg. molar mass ( $\bar{M}_m$ )

Obtained by the carrying out summation over the fraction by their corresponding molar mass, i.e.

$$\bar{M}_m = \sum_i w_i M_i \quad ; \quad \text{it also given by symbol } \bar{M}_w$$

$$\text{Where } w_i = \frac{m_i}{m_{\text{total}}}$$

$\bar{M}_w$  = Weight avg. molar mass

Where  $m_i$  is the mass of the polymer molecules, each having molar mass  $M_i$  and  $m_{\text{total}}$  is the total mass of the sample as

$$m_{\text{total}} = \sum_i m_i$$



Thus,  $\bar{M}_m = \frac{\sum_i m_i \cdot M_i}{\sum_i m_i}$

Again  $m_i = \frac{N_i}{N_A} \cdot M_i$  [since  $m_i$  is the mass per unit vol<sup>m</sup> it may be taken as  $c_i$ ]

So,  $\bar{M}_m = \frac{\sum_i N_i \cdot M_i^2}{\sum_i N_i \cdot M_i}$

$$= \frac{N_1 M_1^2 + N_2 M_2^2 + N_3 M_3^2 + \dots}{N_1 M_1 + N_2 M_2 + N_3 M_3 + \dots}$$

We can also write

$$\bar{M}_m = \frac{\sum_i c_i M_i}{\sum_i c_i} \quad \left\{ \begin{array}{l} \bar{M}_m = \frac{\sum_i N_i \cdot M_i}{\sum_i N_i} = \frac{\sum_i m_i \cdot N_A}{\sum_i \frac{m_i \cdot N_A}{M_i}} \\ = \frac{\sum_i c_i}{\sum_i (c_i/M_i)} \end{array} \right.$$

and  $\bar{M}_n = \frac{\sum_i c_i}{\sum_i (c_i/M_i)}$

iii) Z-avg. molar mass ( $\bar{M}_z$ )

It is related as third power of avg. molecular weight in a polydisperse polymer

i.e.  $\bar{M}_z = \frac{\sum_i N_i \cdot M_i^3}{\sum_i N_i \cdot M_i^2}$

Generally  $\bar{M}_z > \bar{M}_m > \bar{M}_n$

iv) viscosity avg. molar mass ( $\bar{M}_v$ )

The viscosity avg. molar mass is defined as

$$\bar{M}_v = \left( \frac{\sum_i N_i \cdot M_i^{(a+1)}}{\sum_i N_i \cdot M_i} \right)^{1/a}$$

$$\Rightarrow (\bar{M}_v)^a = \frac{\sum_i N_i \cdot M_i^{(a+1)}}{\sum_i N_i \cdot M_i} = \frac{\sum_i c_i \cdot M_i^a}{\sum_i c_i}$$

Where  $a$  is characterised by the system under investigation and generally lies in the range  $0.5 < a \leq 1.0$

Note,  $\bar{M}_v = \bar{M}_m$  for  $a=1$ .

Q. Show that  $\bar{M}_m \gg \bar{M}_n$

Ans

$$\bar{M}_m = \frac{\sum_i N_i M_i^2}{\sum_i N_i M_i} \quad \text{and} \quad \bar{M}_n = \frac{\sum_i N_i M_i}{\sum_i N_i}$$

$$\text{Now, } \bar{M}_m - \bar{M}_n = \frac{\sum_i N_i M_i^2}{\sum_i N_i M_i} - \frac{\sum_i N_i M_i}{\sum_i N_i}$$

$$= \frac{\left\{ \sum_i N_i \cdot \sum_i N_i M_i^2 - \left( \sum_i N_i M_i \right)^2 \right\} \times \frac{1}{\sum_i N_i^2}}$$

$$= \frac{\left\{ \sum_i N_i \cdot \sum_i N_i M_i \right\} \cdot \frac{1}{\sum_i N_i^2}}$$

$$= \frac{\frac{\sum_i N_i M_i^2}{\sum_i N_i} - \left( \frac{\sum_i N_i M_i}{\sum_i N_i} \right)^2}{\frac{\sum_i N_i M_i}{\sum_i N_i}}$$

$$= \frac{\bar{M}_m^2 - \bar{M}_n^2}{\bar{M}_n}$$

$$\text{Now, } (M_i - \bar{M}_n)^2 \geq 0$$

$$\Rightarrow M_i^2 - 2M_i \bar{M}_n + \bar{M}_n^2 \geq 0$$

$$\Rightarrow \bar{M}_n^2 - 2\bar{M}_n^2 + \bar{M}_n^2 \geq 0 \quad \left[ \text{for } i \text{ becoming } n^{\text{th}} \text{ constant} \right]$$

$$\Rightarrow \bar{M}_n^2 - \bar{M}_n^2 \geq 0$$

$$\text{So, } \frac{\bar{M}_n^2 - \bar{M}_n^2}{\bar{M}_n} \geq 0$$

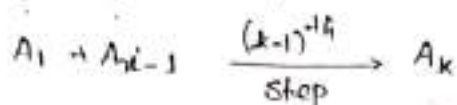
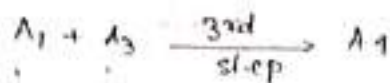
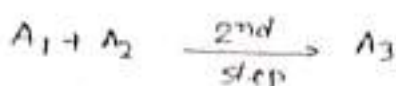
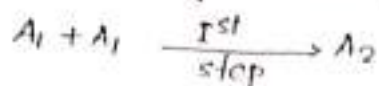
$$\Rightarrow \bar{M}_m - \bar{M}_n \geq 0$$

$$\Rightarrow \bar{M}_m \gg \bar{M}_n$$

Proved -

## Distribution of molecular sizes

Let us take a generalise polymerisation as



Probability of finding a polymer with size 'k' is equal to the probability of having (k-1) elementary step in polymerisation process. i.e. in a k-mer, there are (k-1) no. of group A reacted.

Now probability of each step is equal because

- i) The react<sup>n</sup> does not depend on the chain length and
- ii) Conc. of monomer is very high large.

Thus Probability of k-mer is given by

$$\pi_k \propto p^{(k-1)} \quad ; \quad p = \text{extent of polymerisation}$$

$$\Rightarrow \pi_k = c p^{(k-1)} \quad ; \quad c = \text{Proportionality const.}$$

$$\Rightarrow \sum_{k=1}^{\infty} \pi_k = c \sum_i p^{(k-1)}$$

$$\Rightarrow 1 = c [1 + p^2 + p^3 + \dots] \quad \left[ \text{Since series } (1 + p^2 + p^3 + \dots) \text{ gives } (1-p)^{-1} \right]$$

$$\Rightarrow 1 = c (1-p)^{-1}$$

$$\Rightarrow c = (1-p)$$

$$\text{So, } \pi_k = (1-p) p^{(k-1)}$$

Let us calculate  $\bar{M}_n$  and  $\bar{M}_w$  in terms of P

$$\bar{M}_n = \sum_i \left( \frac{N_i}{N} \right) M_i = \sum_i \pi_i \cdot M_i$$

$$= \sum_i (1-p) p^{(i-1)} \cdot M_i = \sum_i (1-p) \cdot p^{(i-1)} \cdot i M_0$$

$$= (1-p) M_0 \sum_i i p^{(i-1)} \quad \left\{ \begin{array}{l} i = \frac{M_i}{M_0} ; M_0 = \text{mass of monomer} \\ (1-p) = \sum_i (1-p) p^{(i-1)} = (1-p)^{-2} \end{array} \right.$$

$$= (1-p) M_0 [1 + 2p + 3p^2 + \dots]$$

$$= (1-p) M_0 \cdot \frac{1}{(1-p)^2}$$

$$\Rightarrow \bar{M}_n = \frac{M_0}{(1-p)} \quad \Rightarrow \frac{\bar{M}_w}{\bar{M}_n} = \frac{1}{(1-p)}$$



So, no. avg. extent of polymerisation ( $\bar{X}_n$ ) is given as

$$\bar{X}_n = \frac{M_n}{M_0} = \frac{1}{1-p}$$

$$\left\{ \begin{aligned} \sum_{k=0}^{\infty} p^k &= 1 + p + p^2 + \dots = \frac{1}{1-p} \\ \Rightarrow \sum_{k=1}^{\infty} k p^{(k-1)} &= -\frac{1}{(1-p)^2} \cdot (-1) = \frac{1}{(1-p)^2} \end{aligned} \right.$$

Now, we have

$$\bar{M}_n = \sum_i w_i M_i \longrightarrow (1)$$

$$\begin{aligned} \text{Also, } \bar{M}_n &= \frac{\sum_i N_i M_i^2}{\sum_i N_i M_i} = \frac{\sum_i \left(\frac{N_i}{N_0}\right) M_i^2}{\sum_i \left(\frac{N_i}{N_0}\right) M_i} \\ &= \frac{\sum_i \pi_i M_i^2}{\sum_i \pi_i M_i} = \frac{\sum_i \pi_i \cdot i M_0 \cdot M_i}{\sum_i \pi_i \cdot i M_0} \\ &= \frac{\sum_i i \pi_i M_i}{\sum_i i \pi_i} \longrightarrow (2) \end{aligned}$$

Comparing eqn (1) and (2) we get

$$\begin{aligned} w_i &= \frac{i \pi_i}{\sum_i i \pi_i} \\ &= \frac{i (1-p) \cdot p^{(i-1)}}{\sum_i i (1-p) p^{(i-1)}} = \frac{i p^{(i-1)}}{\sum_i i p^{(i-1)}} \end{aligned}$$

$$\Rightarrow w_i = i p^{(i-1)} \cdot (1-p)^2$$

$$\begin{aligned} \text{So, } \bar{M}_n &= \sum_i w_i M_i = \sum_i i (1-p)^2 p^{(i-1)} M_i \\ &= (1-p)^2 \sum_i i \cdot p^{(i-1)} \cdot i M_0 \\ &= (1-p)^2 M_0 \sum_i i^2 p^{(i-1)} \longrightarrow (3) \end{aligned}$$

Let evaluate  $\sum_i i^2 p^{(i-1)}$

we have already

$$\sum_i i^k p^{i-1} = \frac{1}{(1-p)^2}$$

$$\Rightarrow \sum_i i^2 p^i = \frac{p}{(1-p)^2} \quad \left[ \text{multiplying both sides by } p \right]$$

Now differentiating both sides we have

$$\sum_i i^2 p^{i-1} = \frac{1}{(1-p)^2} + \frac{-p}{(1-p)^3} \times (-1)$$

$$= \frac{1-p+2p}{(1-p)^3} = \frac{1+p}{(1-p)^3}$$

Therefore eqn (3) becomes

$$\bar{M}_m = (1-p)^2 \cdot M_0 \cdot \frac{1+p}{(1-p)^3}$$

$$= \frac{1+p}{1-p} M_0$$

$$\Rightarrow \frac{\bar{M}_m}{M_0} = \frac{1+p}{1-p}$$

$$\Rightarrow \bar{M}_m = (1+p) \bar{M}_n$$

$$\Rightarrow \frac{\bar{M}_m}{\bar{M}_n} = (1+p)$$

$$\left[ \begin{array}{l} \bar{M}_m = \frac{M_0}{(1-p)} \\ \frac{\bar{M}_m}{\bar{M}_n} = \text{Polydispersity index} \end{array} \right]$$

We have mass fraction or weight fraction of polymer ( $w_i$ ) as

$$w_i = i (1-p)^2 p^{i-1}$$

$$\Rightarrow \ln w_i = \ln i + 2 \ln(1-p) + (i-1) \ln p$$

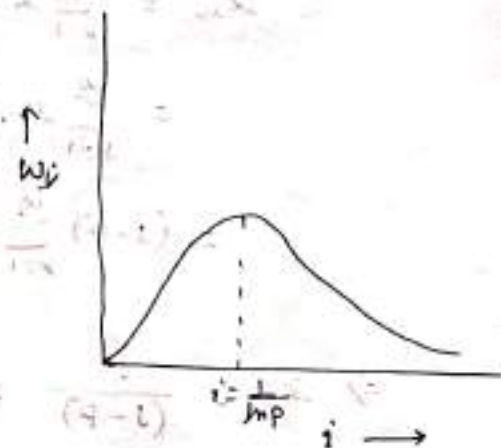
$$\Rightarrow \frac{d \ln w_i}{d i} = \frac{1}{i} + \ln p$$

$$\Rightarrow \frac{1}{w_i} \frac{d w_i}{d i} = \frac{1}{i} + \ln p$$

$$\Rightarrow \frac{d^2 \ln w_i}{d i^2} = -\frac{1}{i^2} < 0 \quad \text{i.e. maxima}$$

Setting  $\frac{d w_i}{d i}$  as zero we get

$$i_{\text{max}} = \frac{1}{\ln p}$$



$$w_i = i (1-p)^2 i p^{(i-1)}$$

$$P_{mi} = M_0 w_i = M_0 (1-p)^2 i p^{(i-1)}$$

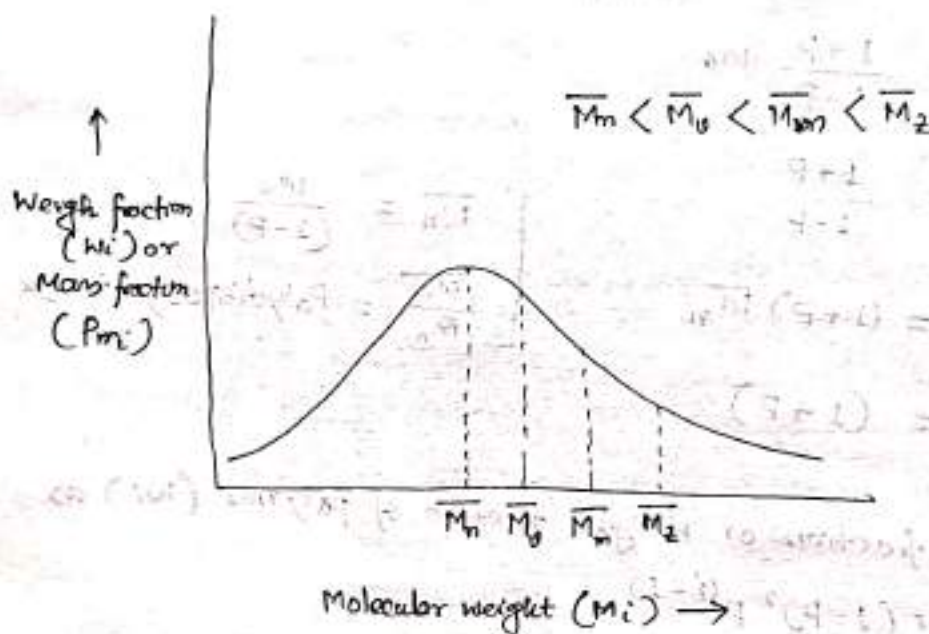
$$= (1-p)^2 M_i p^{i-1}$$

$$= (1-p)^2 M_i p^{\left(\frac{M_i}{M_0} - 1\right)}$$

$$\Rightarrow \ln P_{mi} = 2 \ln(1-p) + \ln M_i + \left(\frac{M_i}{M_0} - 1\right) \ln p$$

$$\Rightarrow \frac{d \ln P_{mi}}{d M_i} = \frac{1}{M_i} + \frac{1}{M_0} \ln p$$

$$\Rightarrow \frac{d^2 \ln P_{mi}}{d M_i^2} = -\frac{1}{M_i^2} < 0$$



Avg. degree of polymerization ( $k_{av}$ )

Avg. value of  $k$  is defined as

$$k_{av} = \sum_{k=1}^{\infty} k \pi_k$$

$$= \sum_{k=1}^{\infty} k \cdot (1-p) p^{(k-1)}$$

$$= (1-p) \sum_{k=1}^{\infty} k \cdot p^{(k-1)}$$

$$= \frac{1-p}{(1-p)^2}$$

$$\Rightarrow k_{av} = \frac{1}{1-p} = \frac{M_p}{M_0}$$



Q) Define Polydispersity index and hence show that  $\overline{M}_w > \overline{M}_n$

Ans The ratio  $\frac{\overline{M}_w}{\overline{M}_n}$  is a measure of the polydispersity of a sample of a polymer. i.e.

$$\text{Polydispersity} = \frac{\overline{M}_w}{\overline{M}_n} = (1+P)$$

where P = extent of polymerisation

$$\overline{M}_w = \frac{\sum_i N_i M_i^2}{\sum_i N_i M_i} \quad \text{and} \quad \overline{M}_n = \frac{\sum_i N_i M_i}{\sum_i N_i}$$

For monodisperse sample, P = 0

$$\text{then } \frac{\overline{M}_w}{\overline{M}_n} = 1 \Rightarrow \overline{M}_w = \overline{M}_n$$

For polydisperse sample, P > 0, then

$$\frac{\overline{M}_w}{\overline{M}_n} > 1 \Rightarrow \overline{M}_w > \overline{M}_n$$

Combining we get  $\overline{M}_w > \overline{M}_n$

Proved.

Q) A hydroxyacid  $\text{HO}-(\text{CH}_2)_5-\text{COOH}$  is polymerised and is found that the pdt has a no. avg. molar mass 20000 g/mol.

i) What is extent of reaction? ii) What is the degree of polymerisation?

iii) Find the mass avg. molar mass?

Ans

i) We have  $\overline{M}_n = \frac{M_0}{1-P}$

$$\Rightarrow P = 1 - \frac{M_0}{\overline{M}_n} = 1 - \frac{132}{20000}$$

$$= 1 - 0.0066 = 0.9934$$

ii)  $k_{av} = \frac{1}{(1-P)}$

$$= \frac{1}{1-0.9934} = 151.5$$

$$k_{av} = \frac{\overline{M}_n}{M_0} = \frac{20000}{132} = 151.5$$

iii)  $\overline{M}_w = \overline{M}_n (1+P)$

$$= 20000 (1+0.9934) \text{ g/mol}$$

$$= 39868 \text{ g/mol}$$

Q. A solution contains equal no of particles with molar masses 10000 g/mol and 20000 g/mol respectively. Calculate  $\overline{M}_n$  and  $\overline{M}_m$ .

Ans

$$\overline{M}_n = \frac{N_1 M_1 + N_2 M_2}{N_1 + N_2}$$

Since  $N_1 = N_2$ , we get

$$\overline{M}_n = \frac{M_1 + M_2}{2} = \frac{10000 + 20000}{2} \text{ g/mol} = 15000 \text{ g/mol}$$

Again  $\overline{M}_m = \frac{N_1 M_1^2 + N_2 M_2^2}{N_1 M_1 + N_2 M_2}$

$$= \frac{M_1^2 + M_2^2}{M_1 + M_2} \quad [\because N_1 = N_2]$$

$$= \frac{(10000)^2 + (20000)^2}{10000 + 20000} \text{ g/mol} = 16666.6 \text{ g/mol}$$

Q. A suspension contains equal masses of particles with molar masses 10000 g/mol and 20000 g/mol respectively. Calculate  $\overline{M}_n$  and  $\overline{M}_m$ .

Ans

Since,  $m = \frac{N}{N_A} \cdot M$ , so,  $N = \left(\frac{m}{M}\right) N_A$

$$\overline{M}_n = \frac{N_1 M_1 + N_2 M_2}{N_1 + N_2} = \frac{\frac{m_1}{M_1} N_A M_1 + \frac{m_2}{M_2} N_A M_2}{\left(\frac{m_1}{M_1}\right) N_A + \left(\frac{m_2}{M_2}\right) N_A}$$

$$= \frac{m_1 + m_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}} = \frac{2m_1}{m_1 \left(\frac{1}{M_1} + \frac{1}{M_2}\right)} \quad [\because m_1 = m_2]$$

$$= \frac{2 M_1 M_2}{M_1 + M_2} = \frac{2 \times 10000 \times 20000}{20000 + 20000} \text{ g/mol} = 13333.3 \text{ g/mol}$$

Again,  $\overline{M}_m = \frac{N_1 M_1^2 + N_2 M_2^2}{N_1 M_1 + N_2 M_2} = \frac{N_A m_1 M_1 + N_A m_2 M_2}{N_A m_1 + N_A m_2}$

$$= \frac{m_1 M_1 + m_2 M_2}{m_1 + m_2} = \frac{m(M_1 + M_2)}{2m_1} \quad [\because m_1 = m_2]$$

$$= \frac{M_1 + M_2}{2} = \frac{10000 + 20000}{2} \text{ g/mol}$$

$$= 15000 \text{ g/mol}$$



## Molecular weight determination

- i) Osmometry
- ii) Viscometry
- iii) Light scattering
- iv) End-group analysis
- v) Sedimentation, etc.

### Osmometry

The no. avg. molar mass of many polymer can be measured conveniently by osmometry. Using the colligative nature of osmotic pressure, the value of  $\bar{M}_n$  is determined.

From thermodynamic derivation of osmotic pressure ( $\pi$ ) we have

$$\pi V = nRT = \frac{c}{M} RT \Rightarrow \pi = \frac{c}{M} RT$$

$V = \text{Vol}^m$  of sol<sup>m</sup>,  $T = \text{temp}$  of experiment

$n = \text{no. of moles of species}$ .

The above eq<sup>n</sup> is strictly valid for ideal and monodisperse sol<sup>n</sup> only. Since polymer is a non-ideal sol<sup>n</sup> and also polydisperse. So, it will not directly applicable for polymer sol<sup>n</sup>.

Thus, with the introduction of non-ideality  $\pi$  may be written as

$$\pi = \frac{c}{M} RT + b(T) \cdot RT \left( \frac{c}{M} \right)^2 + \dots \rightarrow (1)$$

For ideal sol<sup>n</sup>  $b(T) = 0$

$$\text{Now } c = \frac{m}{V} = \frac{N}{V}$$

$$\text{Now, } \frac{c}{M} = \frac{n}{V} = \frac{m}{VM}$$

$$= \frac{N}{N_A V} \Rightarrow c = \frac{NM}{N_A V} \rightarrow (2)$$

Thus rearranging we get

$$\pi = \frac{N}{N_A V} RT + b(T) \cdot RT \cdot \left( \frac{N}{N_A V} \right)^2 + \dots \rightarrow (3)$$

Again from eq<sup>n</sup> (2) we have

$$c = \frac{NM}{N_A V}$$

$$\therefore c_1 = \frac{N_1 M_1}{N_A V}, c_2 = \frac{N_2 M_2}{N_A V}, \dots, c_i = \frac{N_i M_i}{N_A V} \text{ and so on}$$



Thus introducing polydispersity - here we have

$$e = \sum_i c_i$$

$$= \frac{\sum_i N_i \cdot M_i}{N_A V} = \frac{\sum_i N_i \cdot M_i}{\sum_i N_i} \cdot \frac{\sum_i N_i}{N_A V}$$

$$= \frac{M_n \cdot N}{N_A \cdot V}$$

$$\Rightarrow \frac{e}{M_n} = \frac{N}{N_A V} \rightarrow (1)$$

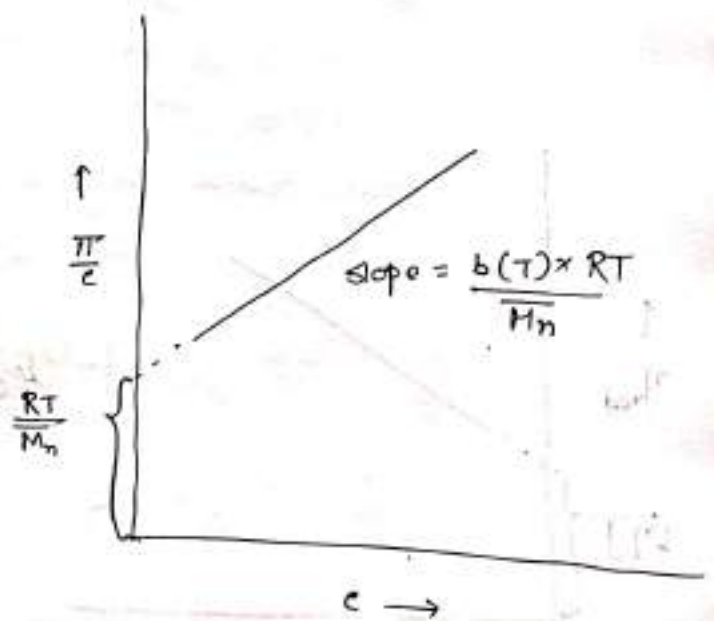
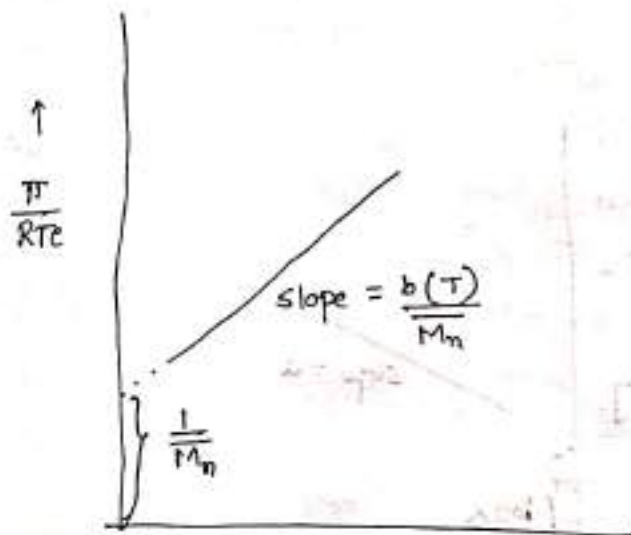
substitute (1) in eqn (3) we may write

$$\pi = \frac{e}{M_n} RT + b(T) \left( \frac{e}{M_n} \right)^2 RT + \dots$$

$$\Rightarrow \frac{\pi}{cRT} = \frac{1}{M_n} + b(T) \cdot \frac{e}{M_n} \quad \left\{ \text{neglect all the higher terms} \right\}$$

$$\Rightarrow \frac{\pi}{e} = \frac{RT}{M_n} + b(T) \cdot \frac{RTc}{M_n}$$

The above eqn are applicable for polymer sol<sup>n</sup>.



$$\left( 1 - \frac{1}{M_n} \right) \frac{1}{c} = \frac{RT}{M_n} + b(T) \cdot \frac{RTc}{M_n}$$

## Viscometry

The most evident distinguishing property of a polymer sol<sup>n</sup> is its viscosity. The measurement provide viscosity avg. molar mass ( $\bar{M}_v$ ). Determination of  $\bar{M}_v$  requires the measurement of the viscosity of a polymer sol<sup>n</sup> relative to that of the pure solvent.

The intrinsic viscosity of a polymer sol<sup>n</sup> is found to increase with increase in avg. molar mass of polymer ( $\bar{M}_v$ ).

This is given by Mark-Houwink eq<sup>n</sup> as

$$[\eta] = k \bar{M}_v^a$$

where  $k$  and  $a$  are const

Taking logs we get

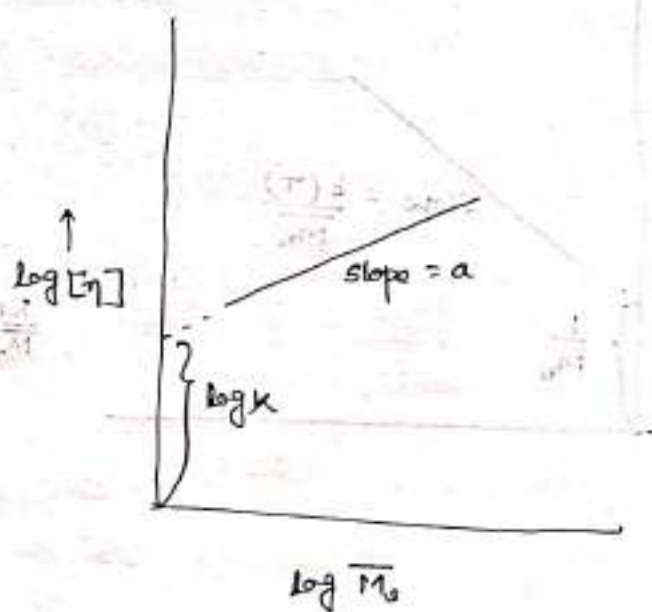
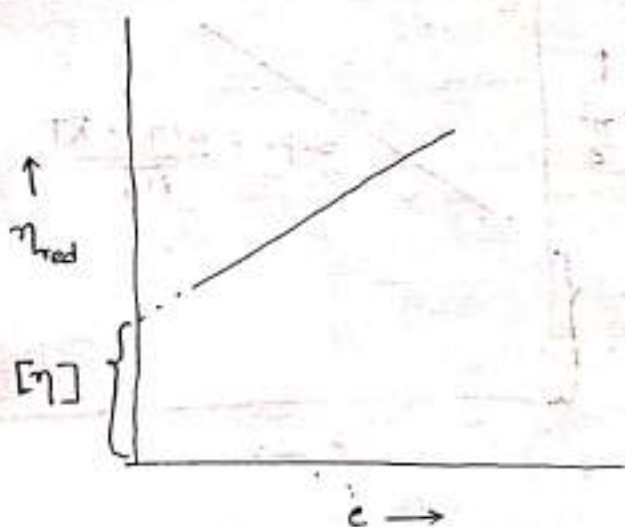
$$\log [\eta] = \log k + a \log \bar{M}_v$$

The value of  $k$  and  $a$  depends on the nature of polymer, nature of the solvent and temp.

Knowing the value of  $a$  and  $k$  from plot of  $\log [\eta]$  vs  $\bar{M}_v$

The value of  $\bar{M}_v$  can be determined.

The value of  $[\eta]$  is determined from the intercept of two graphs of  $\eta_{red}$  vs  $c$



$$\eta_{red} = \lim_{c \rightarrow 0} \frac{\eta_{sp}}{c} = \lim_{c \rightarrow 0} \frac{1}{c} \left( \frac{\eta}{\eta_0} - 1 \right)$$

$$\eta = \eta_0 \left( 1 + c'c + \dots \right)$$

The avg molar mass obtained from viscometry is the  $\bar{M}_v$ . This follows from the following analysis

Since  $[\eta]$  is a limiting value as  $c \rightarrow 0$ , the conc. effect it contains can be written as

$$[\eta] = \lim_{c \rightarrow 0} \frac{\eta_{sp}}{c}$$

For a polydisperse system, we get

$$\begin{aligned} \eta_{sp} &= \sum_i \eta_{sp,i} \\ &= \sum_i c_i [\eta]_i = \sum_i c_i (k M_i^a) \quad \left[ \because [\eta] = k \bar{M}^a \right] \end{aligned}$$

Again  $c = \sum_i c_i$

Hence  $[\eta] = \lim_{c \rightarrow 0} \frac{\sum_i c_i (k M_i^a)}{\sum_i c_i}$

$$\Rightarrow k \bar{M}^a = \frac{k \sum_i c_i M_i^a}{\sum_i c_i}$$

$$\Rightarrow \bar{M}^a = \frac{\sum_i c_i M_i^a}{\sum_i c_i}$$

$$= \frac{\sum_i N_i M_i^{(a+1)}}{\sum_i N_i M_i} = \bar{M}_0$$

$$\Rightarrow \bar{M} = \left[ \frac{\sum_i N_i M_i^{(a+1)}}{\sum_i N_i M_i} \right]^{1/a} = \bar{M}_0^{1/a}$$

For  $a=1$ ,  $\bar{M}_0 = \bar{M}_w$

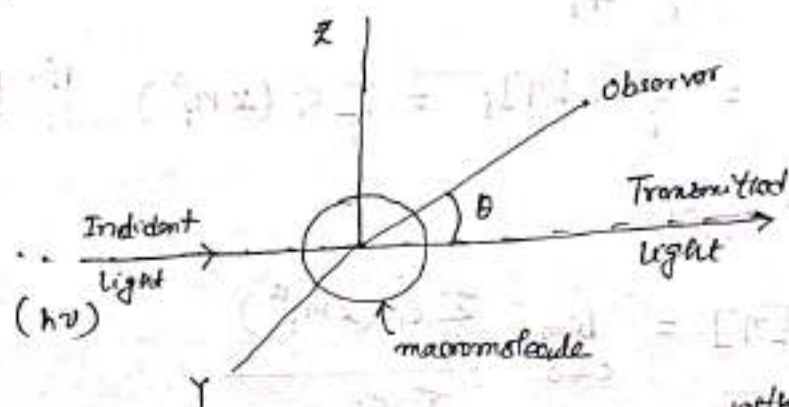
For most polymer system, the value  $a$  lies in the range of 0.5 to 0.8



## Light Scattering techniques

If a light beam is passed through a colloidal sol<sup>n</sup>, it is possible to see the light beam from the sides. This is the well known Tyndal effect which results from the scattering of a part of beam of light by the colloidal particles in all direction.

In case of macromolecules, the colloidal particles are highly solvated and the gel particles consist of about 90 to 99% of solvent. The refractive index of such particles is not very different from that of the solvent.



According to Debye eq<sup>n</sup> turbidity ( $\gamma$ ) of a system, is given as

$$\gamma = \frac{32\pi^3}{3\lambda^4} \left( n \frac{d\eta}{dc} \right)^2 \cdot \frac{k_B T c}{\left( \frac{d\pi}{dc} \right)} \quad \text{with conc. } c \rightarrow (1)$$

where  $\left( \frac{d\pi}{dc} \right)$  is the conc. dependence of eq<sup>m</sup> osmotic pressure

$\lambda$  = wavelength of the incident light

$n$  = refractive index of medium

Now, we know osmotic pressure ( $\pi$ ) is given as

$$\pi = RT \left[ \frac{c}{M} + b(T) \left( \frac{c}{M} \right)^2 + \dots \right]$$

$$\Rightarrow \frac{d\pi}{dc} = RT \left[ \frac{1}{M} + 2 \cdot b(T) \cdot \frac{c}{M^2} + \dots \right]$$

$$= RT \left[ \frac{1}{M} + 2Bc + \dots \right] \quad \left[ B = \frac{b(T)}{M^2} \right]$$

$$\Rightarrow \frac{d\pi}{dc} = RT \left( \frac{1}{M} + 2Bc \right) \quad \text{[neglecting higher terms]}$$

Putting  $\left( \frac{d\pi}{dc} \right)$  in eq<sup>n</sup> (1) we get

$$\gamma = \frac{32\pi^3}{3\lambda^4} \left( n \frac{d\eta}{dc} \right)^2 \cdot \frac{k_B T c}{RT \left( \frac{1}{M} + 2Bc \right)}$$

$$\Rightarrow \gamma = \frac{32\pi^3}{9\lambda^4} \left( n \frac{dn}{dc} \right)^2 \cdot \frac{1}{N_A} \cdot \frac{c}{\frac{1}{M} + 2Bc}$$

$$\Rightarrow \gamma = \frac{Hc}{\frac{1}{M} + 2Bc} \quad \rightarrow (2)$$

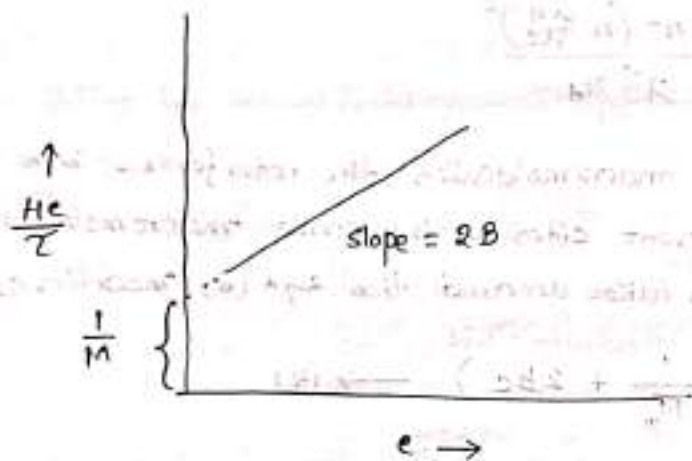
where  $H = \frac{32\pi^3}{9\lambda^4} \left( n \frac{dn}{dc} \right)^2 \cdot \frac{1}{N_A}$  = Debye factor

Inverting eq<sup>n</sup> (2) we get

$$\frac{1}{\gamma} = \frac{\frac{1}{M} + 2Bc}{Hc}$$

$$\Rightarrow \frac{Hc}{\gamma} = \frac{1}{M} + 2Bc \quad \rightarrow (3)$$

Thus a plot of  $\frac{Hc}{\gamma}$  vs  $c$  is a straight line with intercept  $\frac{1}{M}$



For polydisperse system, the molar mass is computed as below

$$c = \sum_i c_i \quad \text{and} \quad \gamma = \sum_i \gamma_i$$

$$\text{Thus } M = \frac{c}{Hc} = \frac{\sum_i \gamma_i}{H \sum_i c_i} \quad [\text{when } c \rightarrow 0 \text{ from plot}]$$

$$\Rightarrow M = \frac{\sum_i H c_i \cdot M_i}{H \sum_i c_i} = \frac{\sum_i c_i \cdot M_i}{\sum_i c_i}$$

$$\Rightarrow M = \frac{\sum_i N_i \cdot M_i^2}{\sum_i N_i \cdot M_i} = \overline{M}_m \quad \rightarrow (4)$$

Thus introducing polydispersity it is seen that  $M$  obtained in this plot is mass avg. molar mass ( $\overline{M}_m$ )

Substitute eq<sup>n</sup> (4) in (5) we have

$$\frac{Kc}{\tau} = \frac{1}{M_m} + 2Bc \quad \rightarrow (5)$$

Now  $\tau$  related to optical-molar factor  $R(\theta)$  as or  $R_\theta$  as

$$\tau = R_\theta = \frac{3\tau}{16\pi}$$

Hence eq<sup>n</sup> (5) becomes

$$\frac{Kc}{\frac{3\tau}{16\pi} R_\theta} = \frac{1}{M_m} + 2Bc$$

$$\Rightarrow \frac{Kc}{R_\theta} = \frac{1}{M_m} + 2Bc \quad \rightarrow (6)$$

Where  $K$  is known as scattering const i.e.

$$K = \frac{3H}{16\pi} = \frac{2\eta^2 \left( n \frac{dn}{dc} \right)^2}{\lambda^4 N_A}$$

For a sol<sup>n</sup> containing macromolecules, the interference b/w the scattering light from different sites of the same macromolecules cannot be excluded. If this is taken account, then eq<sup>n</sup> (6) modifies as

$$\frac{Kc}{R_\theta} = \frac{1}{P_\theta} \left( \frac{1}{M_m} + 2Bc \right) \quad \rightarrow (7)$$

Where  $P_\theta = 1 + \frac{16\pi^2}{3\lambda^2} \overline{r_g^2} \sin^2 \frac{\theta}{2}$

Where  $\overline{r_g^2} = \text{radii of gyration} = \frac{\eta L^2}{6}$

$\theta$  is the angle b/w scattered and transmitted light  
then now we can write

$$\frac{Kc}{R_\theta} = \left( 1 + \frac{16\pi^2}{3\lambda^2} \overline{r_g^2} \sin^2 \frac{\theta}{2} \right)^{-1} \left( \frac{1}{M_m} + 2Bc \right) \quad \rightarrow (8)$$

$$= \frac{1}{M_m} + S \cdot \sin^2 \frac{\theta}{2} + 2Bc + \dots$$

Where  $S = \frac{16\pi^2}{3\lambda^2} \overline{r_g^2} \cdot \frac{1}{M_m}$



From the eq<sup>n</sup> (8), it is apparent that  $\frac{Kc}{R_\theta}$  is linearly depend on  $\cos^2 \theta$  (for a const.  $c$ ) and also on  $\sin^2 \frac{\theta}{2}$  (provided  $c$  is held const.).

Zimm plot is the representation of light-scattering data which can be used to conduct the following extrapolation on a single graph

- i) In limit,  $c \rightarrow 0$  and  $\theta = 0$ ,  $\left(\frac{Kc}{R_\theta}\right)_{c=0, \theta=0} = \frac{1}{M_w}$
- ii) In limit  $\theta = 0$ ,  $\left(\frac{Kc}{R_\theta}\right)_{\theta=0} = \frac{1}{M_w} + 2Bc$
- iii) In limit  $c = 0$ ,  $\left(\frac{Kc}{R_\theta}\right)_{c=0} = \frac{1}{M_w} + S \cdot \sin^2 \frac{\theta}{2}$

### End group analysis

The number avg. molecular weight ( $\bar{M}_n$ ) of linear polymers can be determined by estimating the no. of end groups by chemical analysis. For the successful application of the end group method, the number and nature of end groups per polymer molecule should be reliably known. This method is applicable to linear condensation polymer and also to add<sup>n</sup> polymers. For example carboxyl terminated polybutadiene, hydroxyl terminated polybutadiene; for detecting and analyzing qualitatively functional end-group of linear polymer, the amino end groups of nylon dissolved in m-cresol are readily determined by titrimetric method. From the knowledge of functional group equivalent and functionality, the molecular weight can be calculated using the following eq<sup>n</sup>

$$\bar{M}_n = \frac{\text{Functionality}}{\text{functional group eqv.}}$$

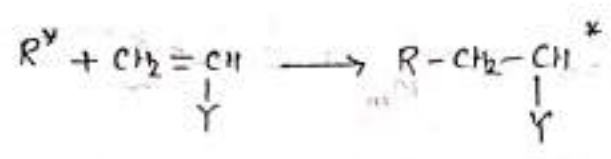
This method is limited to the determination of polymers with a molecular weight of less than 25,000.



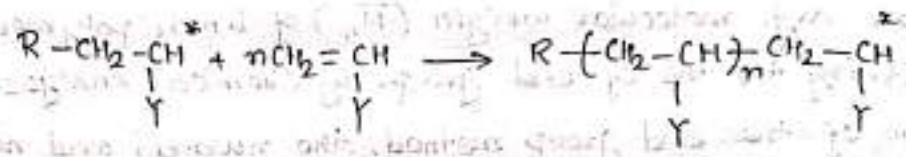
## Kinetics of add<sup>n</sup> polymerization

Add<sup>n</sup> polymerization has the features of a chain reac<sup>n</sup>. The reactive centres may be formed by reac<sup>n</sup> with a free radical, a cation or an anion. Polymerization occurs through the propagation of the active species by the successive add<sup>n</sup> of a large no. of monomer molecules in a chain reac<sup>n</sup> occurring in a matter of a second or so at most, and usually in a much shorter time. By far the most common example of chain polymerization is that of the polymerization of vinyl monomers. The process can be as

### Initiation



### Propagation

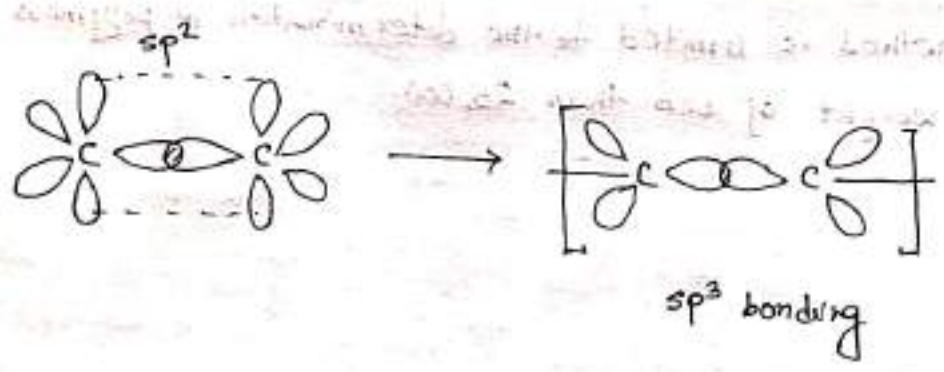
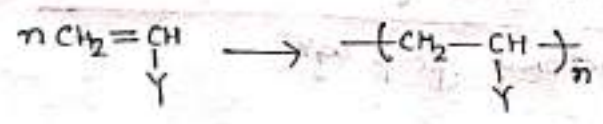


The growth of the polymer chain ceases when the reac<sup>n</sup> centre is destroyed by one of a no. of possible termination reac<sup>n</sup>. The

basic individual steps in the process are thus

- i) Initiation (through UV ray / hv /  $\alpha$ -ray etc.)
- ii) Propagation
- iii) Termination

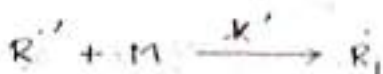
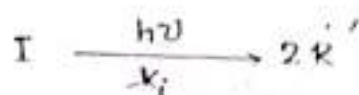
The driving force for the reac<sup>n</sup> is provided by the relief in the steric strain by the change of sp<sup>2</sup> bonding to sp<sup>3</sup> bonding





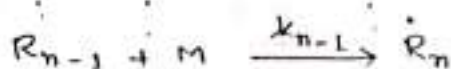
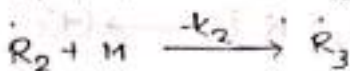
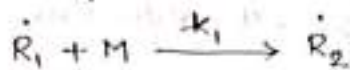
# Mechanism and kinetics of free radical add<sup>n</sup>

i) Initiation

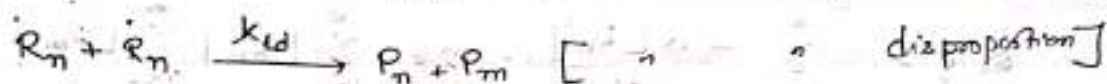
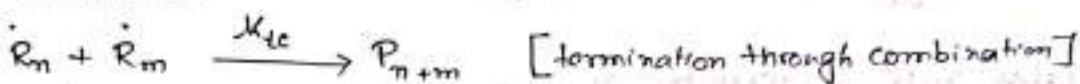


$\begin{cases} I \equiv \text{initiator} \\ R \equiv \text{radical} \\ M \equiv \text{monomer} \end{cases}$

ii) Chain propagation



iii) Chain termination



Now, rate of initiation will be

$$r_i = 2k_i [I] \quad \text{--- (1)}$$

$$r_i = \frac{d[R_1]}{dt} = 2fk_i [I] = k' [R'] [M] \quad \text{--- (2)}$$

Assuming 'f' fraction of monomer is reacted at time t

Again total rate of propagation ( $r_p$ ) will be

$$r_p = k_{1p} [R_1] [M] + k_{2p} [R_2] [M] + \dots$$

Rate of propagation step does not depend on the chain length or size of the radical. so,

$$k_{1p} = k_{2p} = k_{3p} = \dots = k_p$$

Putting this we get

$$r_p = k_p [M] \{ [R_1] + [R_2] + [R_3] + \dots \}$$

$$\Rightarrow r_p = k_p [M] \sum_{n=1}^{\infty} [R_n] \quad \text{--- (3)}$$

Now, Rate of polymerization ( $r_p$ )

= rate of consumption of monomer

$$\text{i.e. } r = - \frac{d[M]}{dt}$$



Since consumption of monomers actually occurs in initiation and propagation.

Thus,  $r = r_i + r_p$

$$= k' [M] [\dot{R}'] + k_p [M] \sum_{n=1}^{\infty} [\dot{R}_n]$$

In the above eq<sup>n</sup> propagation terms have infinity no. of terms. So, neglecting the  $r_i$  term and adding  $n=0$ th term, so that we get all the monomer unit without loss of any; it reduces as

$$r = k_p [M] \sum_{n=0}^{\infty} [\dot{R}_n] = k_p [M] [\dot{R}] \rightarrow (4)$$

From the termination steps

$$- \frac{d[\dot{R}_n]}{dt} = 2k_t [\dot{R}_n] \sum_{m=0}^{\infty} [\dot{R}_m] \quad ; \quad k_t = \frac{k_{tc} + k_{td}}{2}$$

Total rate of termination

$$\begin{aligned} R_t r_t &= \sum_{n=0}^{\infty} - \frac{d[\dot{R}_n]}{dt} = 2k_t \sum_{n=0}^{\infty} [\dot{R}_n] \sum_{m=0}^{\infty} [\dot{R}_m] \\ &= 2k_t [\dot{R}] [\dot{R}] = 2k_t [\dot{R}]^2 \rightarrow (5) \end{aligned}$$

Introducing steady state approximation

$$\frac{d[\dot{R}']}{dt} = \frac{d[\dot{R}_i]}{dt} = 0$$

i.e.  $\frac{d}{dt} \left\{ \sum_{n=0}^{\infty} [\dot{R}_n] \right\} = 0 \Rightarrow \frac{d[\dot{R}]}{dt} = 0$

For steady state approximation

$$r_i = r_t$$

$$\Rightarrow 2fk_i [I] = 2k_t [\dot{R}]^2$$

$$\Rightarrow [\dot{R}] = \left\{ \frac{fk_i [I]}{k_t} \right\}^{1/2} \rightarrow (6)$$

substituting eq<sup>n</sup> (6) in (4) we get

$$\begin{aligned} r &= k_p [M] \left\{ \frac{fk_i [I]}{k_t} \right\}^{1/2} \\ &= k_p \left( \frac{fk_i}{k_t} \right)^{1/2} [I]^{1/2} [M] \end{aligned}$$

## Factors affecting chain-polymerization

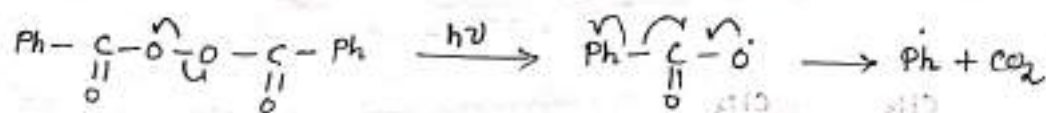
Ethylene,  $\text{CH}_2=\text{CH}_2$  is the only aliphatic hydrocarbon which polymerizes well under free-radical mode of initiation. Propylene and isobutylene are unreactive under these conditions. Electron density at the double bond may determine whether a particular monomer polymerizes by anionic, cationic or free-radical mechanism.

Monomer str.	Mode of initiation
$\text{CH}_2=\overset{\text{X}}{\text{C}}$	Cationic <sup>+</sup> , e.g. butene, styrene, vinyl ether
$\text{CH}_2=\overset{\text{X}}{\text{C}}$	Anionic <sup>-</sup> , e.g. acrylonitrile, styrene
$\text{CH}_2=\overset{\text{X}}{\text{C}}$	Free radical, ethyl acrylate

## Initiators

A no. of polymerization are initiated by heat and UV radiation. Usually, the initiation is peroxide, hydroperoxide, an azo-nitrile, etc, which can readily be converted to free radical under the conditions for the desired polymerization.

One typical initiator is benzoyl peroxide



## Ionic add<sup>n</sup> polymerization

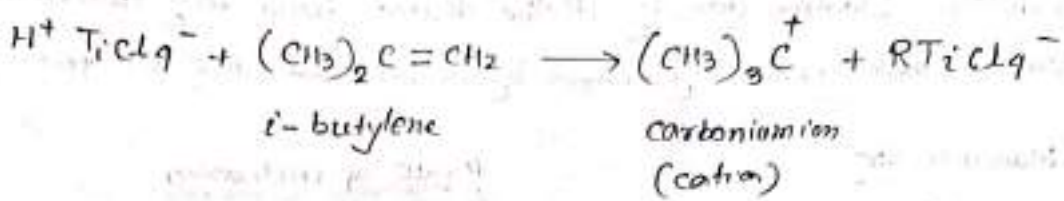
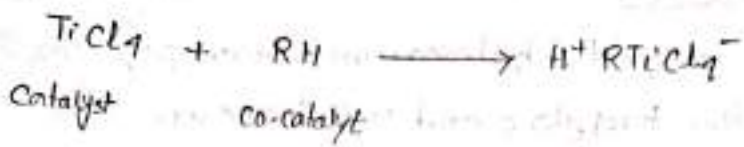
The polymerization of vinyl monomers can take place in the presence of a cation or an anion also.

## Cationic Polymerization

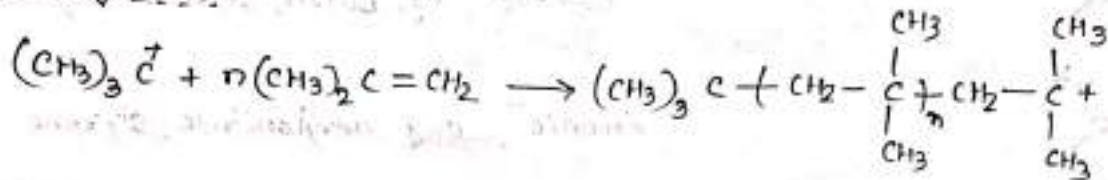
Cationic polymerization differs from free-radical polymerization, specially in the initiation and termination steps. The initiator used are Lewis acid such as  $\text{BF}_3$ ,  $\text{AlCl}_3$ ,  $\text{TiCl}_4$ ,  $\text{H}_2\text{SO}_4$  and other strong protonic acid. All these, except possibly the strong protonic acids, need a co-catalyst to initiate the polymerization. The co-catalyst supplies hydrogen, and the real initiator is believed to be a proton in every case. This means that one end of the polymer is hydrogen atom. The monomers which best polymerize by cationic initiation are olefins, substituted with electron releasing group (EDG) such as  $\text{CH}_2=\text{C}(\text{CH}_3)_2$ ,  $\text{PhCH}=\text{CH}_2$ , etc.



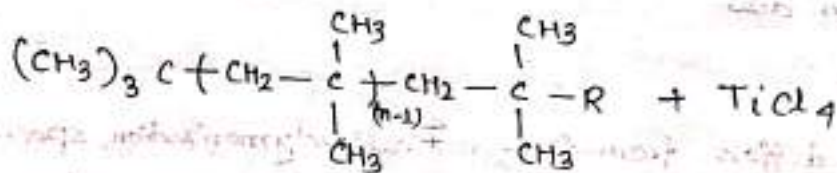
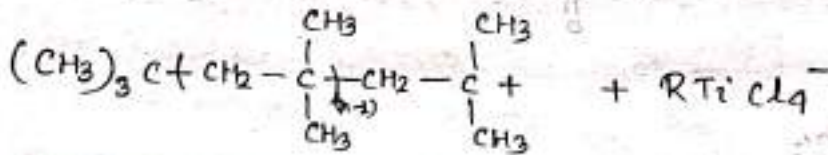
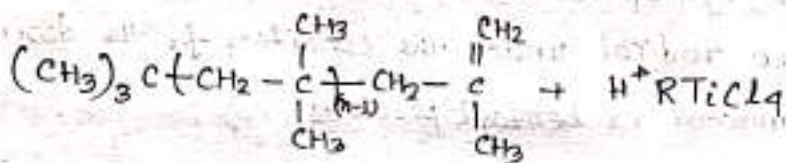
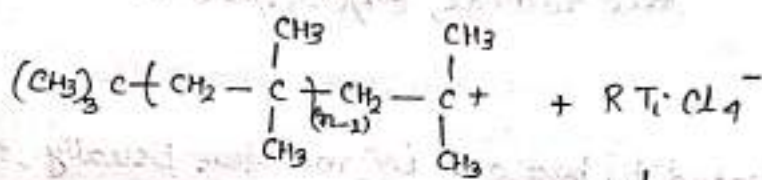
### Initiation



### Propagation



### Termination



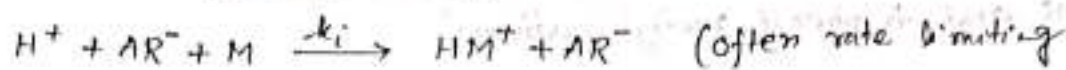
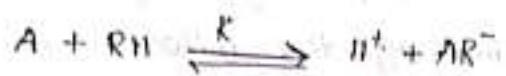
iii. Polymerisation proceeds most rapidly to give the high molecular weight polymers at very low temp. For example *i*-butylene at  $-100^\circ\text{C}$  gives a high polymer in a fraction of second. At room temp a dimer is formed slowly.



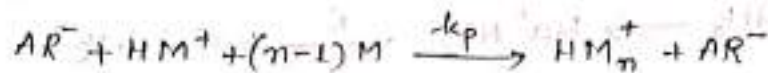
## Kinetics of cationic polymerization

Most of the cationic polymerization proceed so rapidly that it is difficult to establish the steady state. However, the following kinetic scheme seems to be valid, let the catalyst be designated A and co-catalyst by RH. Then,

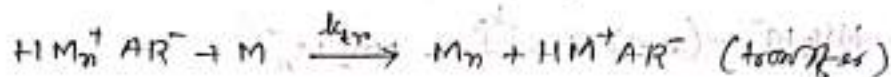
### Initiation



### Propagation



### Termination



Rate of initiation,

$$R_i = K k_i [A][RH][M]$$

If the formation of  $H^+ AR^-$  is the rate determining step,  $R_t$  can be independent of  $[M]$ . The scheme is then appropriately modified. Since termination is first order,

$$R_t = k_t [HM^+ AR^-] = k_t [M^+]$$

Assuming steady state approximation

$$R_i = R_t$$

$$\Rightarrow K k_i [A][RH][M] = k_t [M^+]$$

$$\Rightarrow [M^+] = \frac{K k_i}{k_t} [A][RH][M]$$

Now overall rate of polymerization is decided by the propagation step

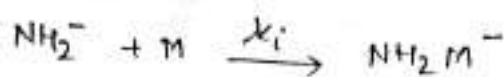
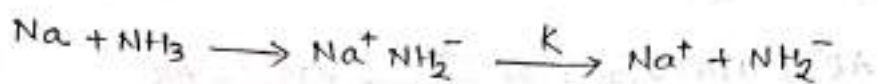
$$\text{i.e. } r = r_p = k_p [M][M^+]$$

$$= K \cdot \frac{k_p k_i}{k_t} [A][RH][M]^2$$

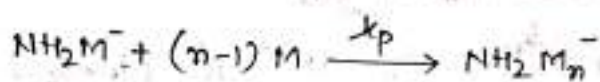
## Anionic polymerization

Monomers containing electron withdrawing group (EWG) or electronegative groups can polymerize by anionic mechanism. It was found that Na in liq.  $\text{NH}_3$  at  $-75^\circ\text{C}$  was a particularly effective initiator for the polymerization of  $\alpha$ -methyl acrylonitrile, methyl acrylate, etc. were polymerized by this mechanism. Other effective initiator are Grignard reagent and triphenyl methyl sodium.

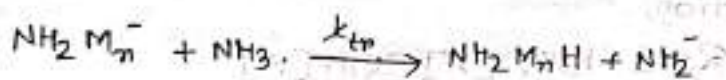
### Initiation



### Propagation



### Termination



Rate of initiation,  $r_i = k_i [\text{NH}_2^-] [\text{M}]$

Applying steady state,  $r_i = r_t$

$$\Rightarrow k_i [\text{NH}_2^-] [\text{M}] = k_{tr} [\text{NH}_2\text{M}_n^-] [\text{NH}_3]$$

$$\Rightarrow [\text{NH}_2\text{M}_n^-] = \frac{k_i}{k_{tr}} \frac{[\text{NH}_2^-]}{[\text{NH}_3]} [\text{M}]$$

Thus rate of polymerization

$$r_p = k_p [\text{NH}_2\text{M}_n^-] [\text{M}]$$

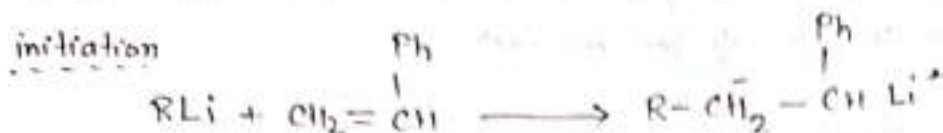
$$= \frac{k_i k_p}{k_{tr}} \frac{[\text{NH}_2^-]}{[\text{NH}_3]} [\text{M}] [\text{M}]$$

$$= \frac{k_i k_p}{k_{tr}} \frac{[\text{NH}_2^-]}{[\text{NH}_3]} [\text{M}]^2$$

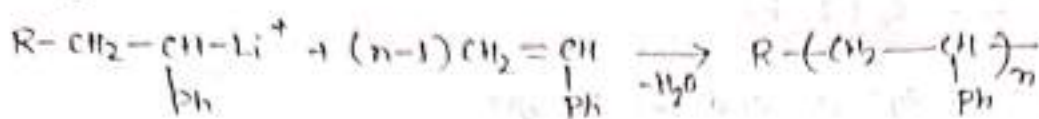
The above is the required rate expression.



The following example show the scope of this type

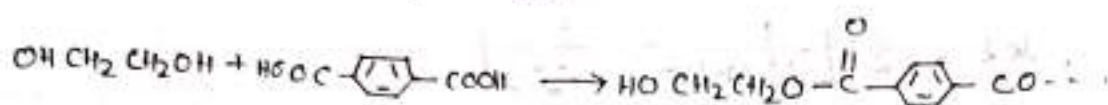


Propagation



Kinetics of condensation (step growth) polymerization

It may be discussed by the example of polyester formation b/w a dicarboxylic acid and ethylene glycol.



The above react<sup>n</sup> may proceed with or without the presence of an acid catalyst.

The acid catalysed rate of polymerisation may be written as

$$-\frac{d[\text{COOH}]}{dt} = k [\text{COOH}] [\text{OH}] [\text{H}^+]$$

$$= k' [\text{COOH}] [\text{OH}] = k' [\text{COOH}]^2 \quad [k' = k [\text{H}^+]]$$

Because  $[\text{COOH}] = [\text{OH}]$  because equimolecular amount of an acid and glycol are generally used. If  $e$  mol/lit be the conc. of acid and glycol

then,  $-\frac{de}{dt} = k' e^2 \Rightarrow 2^{\text{nd}} \text{ order kinetics}$

$$\Rightarrow \int \frac{de}{e^2} = -k' \int dt$$

$$\Rightarrow k' t = \frac{1}{e} - \frac{1}{e_0} \rightarrow (1)$$

When  $t=0$ ,  $e=e_0$   
= initial conc. of reactant

If no external acid is used as catalyst i.e. for uncatalysed react<sup>n</sup>, one molecule of the dicarboxylic acid act as an catalyst. Then,

$$-\frac{d[\text{COOH}]}{dt} = k [\text{COOH}]^2 [\text{OH}]$$

$$\Rightarrow -\frac{de}{dt} = k e^3 \Rightarrow 3^{\text{rd}} \text{ order kinetics}$$

$$\Rightarrow \int \frac{de}{e^3} = -k \int dt$$

$$\Rightarrow k t = \frac{1}{2} \left[ \frac{1}{e^2} - \frac{1}{e_0^2} \right] \rightarrow (2)$$



It is convenient to express the rate of  $p$  in terms of the extent of reaction  $p$ , rather than in terms of conc. of the reactant.

$$\text{Again } p = \frac{c_0 - c}{c_0} = \frac{N_0 - N}{N_0}$$

$$\Rightarrow c = c_0(1-p)$$

Replacing this in eqn (1) and (2) we get

$$k't = \frac{1}{c_0(1-p)} - \frac{1}{c_0}$$

$$\Rightarrow \frac{1}{1-p} = c_0 k't + 1 \quad \text{--- (3)}$$

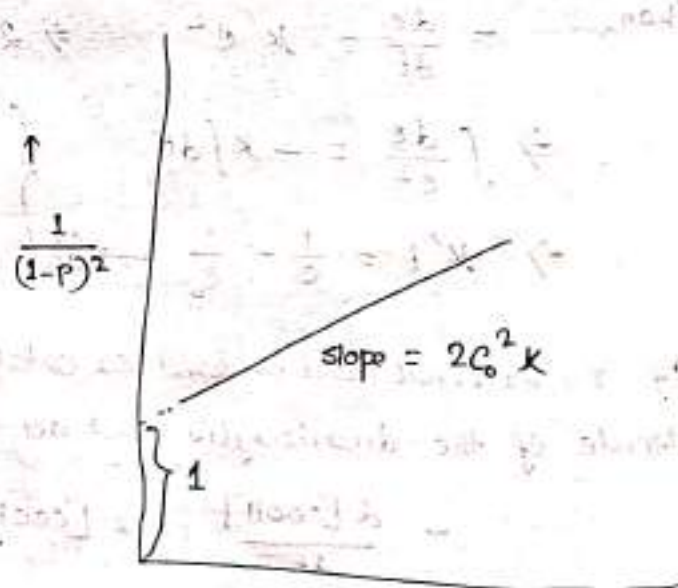
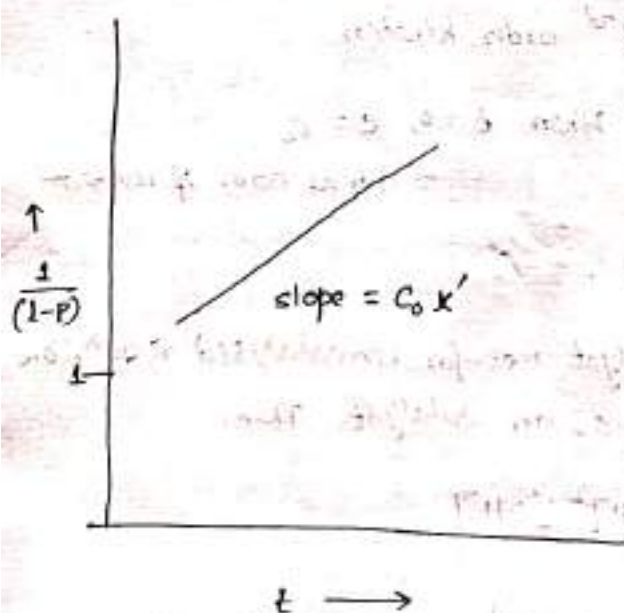
and

$$k't = \frac{1}{2} \left[ \frac{1}{c_0^2(1-p)^2} - \frac{1}{c_0^2} \right]$$

$$\Rightarrow \frac{1}{(1-p)^2} = 2c_0^2 k't + 1 \quad \text{--- (4)}$$

From eqn (3) and (4) it is clear that for acid-catalysed polyesterification, a plot of  $\frac{1}{(1-p)}$  vs  $t$  of polymerization should result a straight line.

Similarly, for uncatalysed polyesterification a plot  $\frac{1}{(1-p)^2}$  vs  $t$  should be linear.

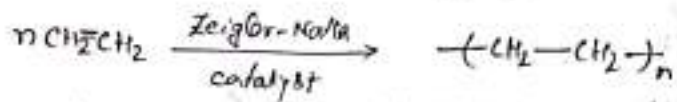


From the slope we can calculate the value of rate const ( $k$ )

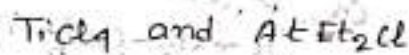
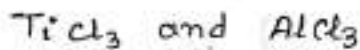
## Co-ordination polymerization

There are a no. of coordination catalyst which can polymerize olefins. The most common catalyst of this type are composed of aluminium trialkyls and titanium or vanadium chloride. These catalyst are mostly effective for the polymerization of 1-olefins.

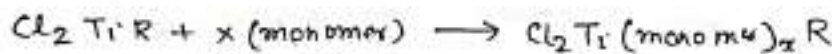
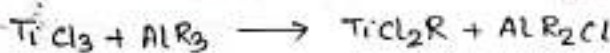
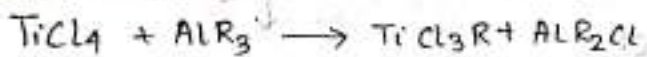
The Ziegler-Natta method, provide a heterogeneous fibrous catalyst suitable for polymerization of olefins (ethylene).



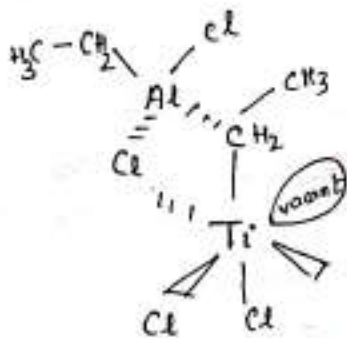
Ziegler discovered that combination of  $\text{TiCl}_4$  and  $\text{AlEt}_2\text{Cl}$  gave comparable activities for the production of polyethylene. Natta used crystalline  $\alpha\text{-TiCl}_3$  in combination with  $\text{AlEt}_3$  to produce first isotactic polypropylene. i.e. two sets are



The following eq<sup>n</sup> seem to best describe the reason b/w titanium tetrachloride and a aluminium trialkyl.



Two type of str. have been put forward for the Ziegler-Natta initiator, a bimetallic (I) and monometallic (II) one.



I

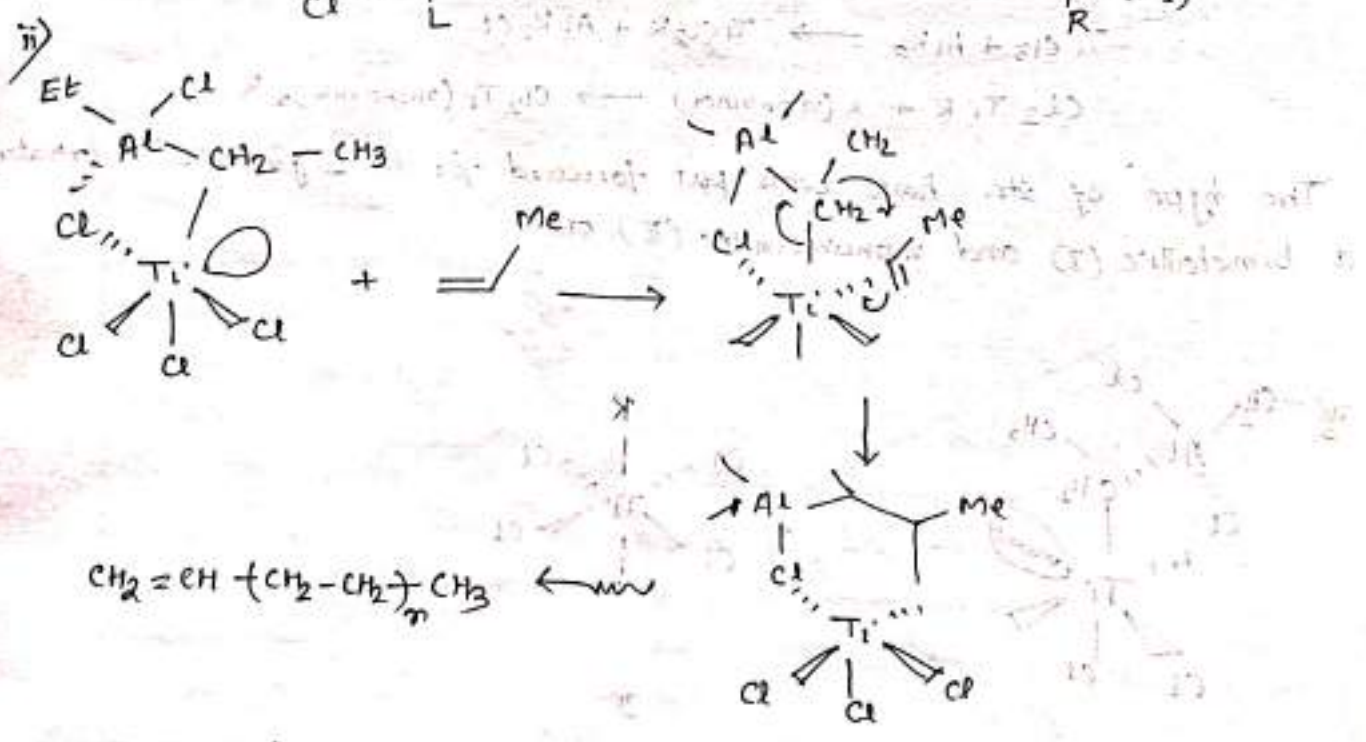
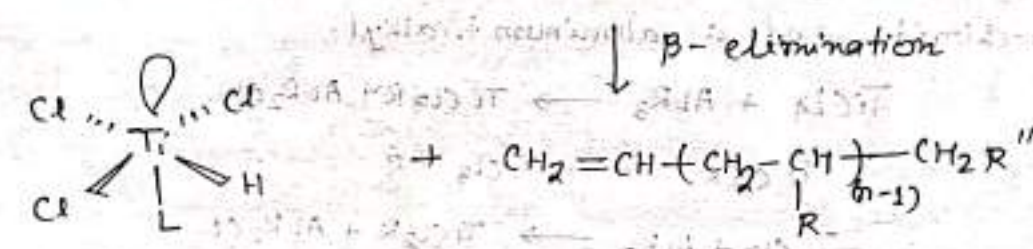
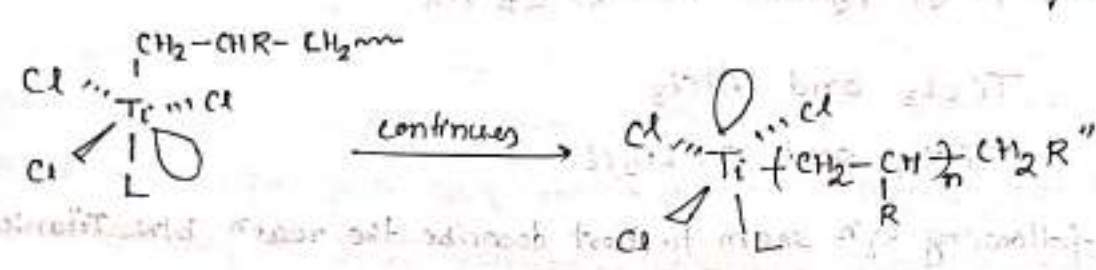
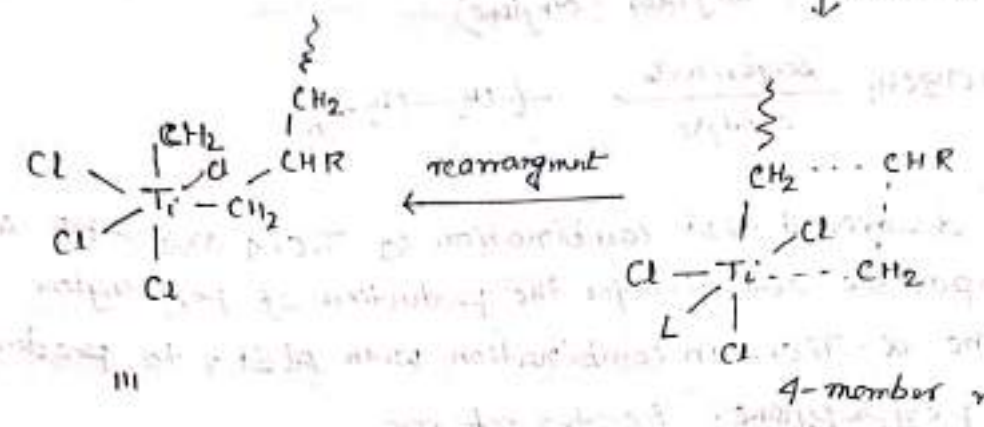
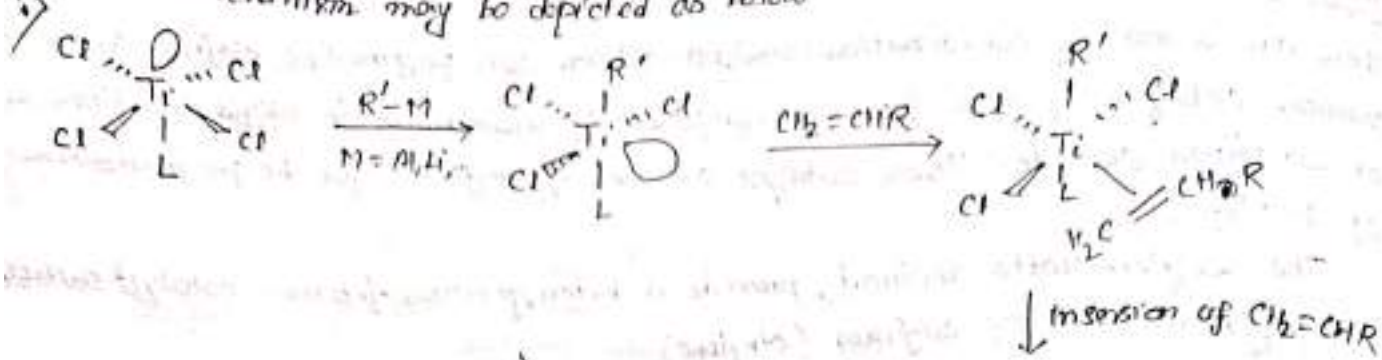


II

L  $\equiv$  unspecified ligand



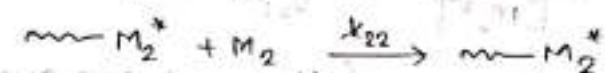
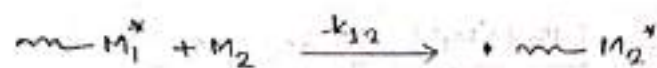
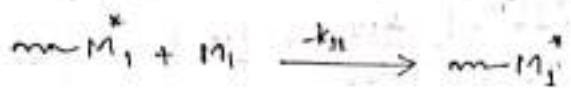
The mechanism may be depicted as below





## Co-polymerization composition eqn

The composition of the co-polymer in most instances is found to be different from that of the co-monomer from which it is produced. In other words, different monomers have different tendencies towards undergoing co-polymerization. Monomers  $M_1$  and  $M_2$  can each add either to propagating chain ending in  $M_1$  or  $M_2$ .



Where  $k_{11}$  is the rate const. for the add<sup>n</sup> of an  $M_1^*$  radical to  $M_1$ ,  $k_{12}$  is the rate const. for the add<sup>n</sup> of an  $M_1^*$  radical to  $M_2$ ,  $k_{21}$  is the rate const. for the add<sup>n</sup> of an  $M_2^*$  of  $M_1$ , and  $k_{22}$  is the rate const. for the add<sup>n</sup> of an  $M_2^*$  to  $M_2$ .

The rate of disappearance of monomer  $M_1$  and  $M_2$  (neglecting any loss in the initiation process) is given by

$$-\frac{d[M_1]}{dt} = k_{11}[M_1^*][M_1] + k_{21}[M_2^*][M_1] \rightarrow (1)$$

$$\text{and } -\frac{d[M_2]}{dt} = k_{12}[M_1^*][M_2] + k_{22}[M_2^*][M_2] \rightarrow (2)$$

Dividing eqn (1) by (2) we get

$$\begin{aligned} \frac{d[M_1]}{d[M_2]} &= \frac{k_{11}[M_1^*][M_1] + k_{21}[M_2^*][M_1]}{k_{12}[M_1^*][M_2] + k_{22}[M_2^*][M_2]} \\ &= \frac{[M_1]}{[M_2]} \cdot \frac{k_{11}[M_1^*] + k_{21}[M_2^*]}{k_{12}[M_1^*] + k_{22}[M_2^*]} \rightarrow (3) \end{aligned}$$

Assuming steady state approximation, we get

$$k_{21}[M_2^*][M_1] = k_{12}[M_1^*][M_2]$$

$$\Rightarrow [M_1] = \frac{k_{12}}{k_{21}} \frac{[M_1^*][M_2]}{[M_2^*]} \rightarrow (4)$$

Now reactivity ratio is given as

$$r_1 = \frac{k_{11}}{k_{12}} \quad \text{and} \quad r_2 = \frac{k_{22}}{k_{21}}$$

Multiplying both numerator and denominator <sup>by factor 2</sup> of RHS of eq<sup>n</sup> (3) we have

$$\frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]} \cdot \frac{\frac{1}{k_{21}} \frac{[M_2]}{[M_2^*]} (k_{11}[M_1^*] + k_{21}[M_2^*])}{\frac{1}{k_{21}} \frac{[M_2]}{[M_2^*]} (k_{12}[M_1^*] + k_{22}[M_2^*])}$$

$$= \frac{[M_1]}{[M_2]} \cdot \frac{\frac{k_{11}}{k_{12}} \frac{[M_1^*][M_2]}{[M_2^*]} \cdot \frac{k_{12}}{k_{21}} + [M_2]}{\frac{k_{12}}{k_{21}} \frac{[M_2][M_1^*]}{[M_2^*]} + \frac{k_{22}}{k_{21}} [M_2]}$$

}  $\alpha = \frac{1}{k_{21}} \frac{[M_2]}{[M_2^*]}$

$$= \frac{[M_1]}{[M_2]} \cdot \frac{[M_1]r_1 + [M_2]}{[M_1] + r_2[M_2]}$$

} Using wrong  $q$  and substitute  $r_1$  and  $r_2$  value replace

$$\Rightarrow \frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]} \cdot \frac{r_1[M_1] + [M_2]}{r_2[M_2] + [M_1]} \quad \text{--- (5)}$$

Eq<sup>n</sup> (5) is called co-polymer composition eq<sup>n</sup>.

### Various polymerization techniques

For laboratory and industrial purposes, polymerization is practiced by a variety of approaches and techniques that require different design considerations w.r.t. recipe of polymerization and, w.r.t. physical conditions for the process and process equipment. Generally, polymerizations are studied and practiced by four different techniques:

- i) Bulk or mass polymerization
- ii) sol<sup>n</sup> polymerization
- iii) suspension polymerization
- iv) Emulsion polymerization

Among all these four techniques, the last two are essentially heterogeneous polymerization systems containing a large portion of a non-solvent, usually water acting as a dispersion medium for the immiscible liquid monomer. Bulk and sol<sup>n</sup> polymerization may be either homogeneous all the way or heterogeneous <sup>homogeneous</sup> to begin with, but becoming heterogeneous with progress of polymerization due to the solvent used to dilute the monomer (for sol<sup>n</sup> polymerization) or polymer formed being insoluble in its monomer (for bulk polymerization). Most condensation polymerizations are conveniently done by employing bulk or sol<sup>n</sup> techniques.



### Bulk polymerization

When a monomer is polymerized in bulk, an initiator which can decompose to give free-radicals at a fairly good rate at the temp of polymerization is used. There are advantages as well as disadvantages of this process. This method is now used for casting objects to be preserved such as an insect, a flower or any other thing. This process can be carried out at comparatively low temp. Poly(methylmethacrylate) sheets and rods are made in this way. However, thermal control is difficult to achieve as also is any further isolation from the monomer.

### Sol<sup>n</sup> polymerization

In sol<sup>n</sup> polymerization, on the other hand, improved thermal control is possible, but it is difficult to remove the solvent. Further, chain-transfer to solvent may limit the molecular weight of polymer obtained. This limits the usefulness of the technique.

### Emulsion and suspension polymerization

Emulsion and suspension polymerization in aqueous media are free from the problems of heat transfer, chain transfer and viscosity. These two methods have assumed industrial importance. It is possible to get higher molecular weights with emulsion polymerization. In emulsion polymerization, the monomer is emulsified in water with an emulsifying agent which may be a soap or detergent. Usually, a redox system is used. In a typical emulsion system a monomer(s), water, emulsifier (surfactant), a chain transfer agent and a water soluble initiator are usually used. ~~This is illustrated by the recipe~~